Physics Factsheet

April 2001 Number 13

# Motion I – Uniform Acceleration

**This Factsheet will cover :**
- **the basic definitions of speed, velocity and acceleration**
- **the use of the equations of motion for uniform acceleration**
- **the application of the equations of motion to projectile motion**

## 1. Basic Concepts

**Displacement** is the distance and direction of a body relative to a given reference point (usually the point at which it starts). As it has both a length and a direction, it is a **vector**. Its SI unit is the **metre** (m). (*eg "200 metres east of my house" is a displacement)*

**Velocity** is the rate of change of displacement. Accordingly, velocity is also a **vector**. Its SI unit is **metres per second** ($ms^{-1}$). (*eg 3 $ms^{-1}$ north is a velocity)*

$$\textbf{Average velocity} = \frac{\textbf{total change in displacement}}{\textbf{time taken}}$$

**Acceleration** is the rate of change of velocity – so, again, is a vector. Its SI unit is **metres per second²** ($ms^{-2}$).

$$\textbf{Average acceleration} = \frac{\textbf{total change in velocity}}{\textbf{time taken}}$$

**Speed and velocity.**
**Speed** is the rate of change of **distance**.

The speed of a body at any instant is equal to the magnitude of its velocity at that instant. However, this is **not** generally the case for average speed and average velocity.

To see why this is, imagine walking 5 m North in 2 s, then 5 m South in 2 s. Since you end up at the point where you started, your overall displacement is zero – so your average **velocity** is **zero**. However, you have travelled a distance of 10 m in 4 s, so your average **speed** is:
10 ÷ 4 = **2.5 $ms^{-1}$**.

The average speed will **only** be the magnitude of the average velocity if the body concerned is moving in a straight line, without reversing its direction – since then, the distance it moves will always be equal to the magnitude of its displacement.

## 2. Equations of motion for uniform acceleration

| | |
|---|---|
| $v = u + at$ | u = initial velocity |
| $v^2 = u^2 + 2as$ | v = final velocity |
| $s = ut + ½at^2$ | a = acceleration |
| $s = ½ (u + v)t$ | s = displacement from the starting point |
| | t = time |

It is important to note when using these equations that u, v, a and s can be negative as well as positive.
- Positive and negative values of displacement (s) refer to positions each side of the starting point – for example, if a positive displacement refers to positions above the starting point, then negative ones will refer to positions below it.
- Positive and negative values of velocity (u or v) refer to its direction. For example, if you throw a ball up in the air, then if its initial velocity – when going up – is positive, then its final velocity – when it is coming down – will be negative.
- Any acceleration that has the **opposite** sign to the **velocity** will act as a **retardation** – in other words it will slow the body down. So, if the velocity is positive, a retardation will be negative.

**Approach to problems using equations of motion**
1. Check that the body is moving with constant acceleration!
2. Write down any of u, v, a, s, t that you know
3. Note down which of u, v, a, s and t that you want (e.g. write a = ?)
4. Decide which equation to use by looking at which of the variables you have got written down in steps 2 and 3. For example, if you have got values for u, t and s, and you want a value for v, then you look for the equation with u, t, s and v in it.
5. Substitute the values you know in, then rearrange.
6. Check that the answer makes sense.

***Example 1. A particle is moving in a straight line with constant acceleration. It passes point P with speed 2 $ms^{-1}$. Ten seconds later, it passes point Q. The distance between P and Q is 40 metres. Find the speed of the particle as it passes point Q.***

*We know:* $u = 2ms^{-1}$ $s = 40$ m $t = 10$ s
*We want:* $v = ?$

*Since we have u, s, t and v involved, use* $s = ½ (u + v)t$
*Substituting in:*
$40 = ½ (2 + v)10$
$40 = 5(2 + v)$
$40 = 10 + 5v$
$30 = 5v$
$v = 6\ ms^{-1}$

***Example 2. A particle is moving in a straight line with constant acceleration 0.2 $ms^{-2}$. After it has moved a total of 20m, its speed is 8$ms^{-1}$. Find its initial speed.***

$a = 0.2\ ms^{-2}$ $s = 20m$ $v = 8\ ms^{-1}$ $u = ?$
*So use* $v^2 = u^2 + 2as$
$8^2 = u^2 + 2(0.2)(20)$
$64 = u^2 + 8$
$56 = u^2$
$u = 7.48\ ms^{-1}$ (3SF)

> ***Tips:***
> 1. *It is usually easier to put the values in* ***before*** *rearranging the equations.*
> 2. *Take particular care with negative values. On many calculators, if you type in $-2^2$, you will get the answer –4, rather than the correct value of 4. This is because the calculator squares before "noticing" the minus sign.*

***Example 3. A particle starts from rest and moves with constant acceleration 0.5 $ms^{-2}$ in a straight line. Find the time it takes to travel a distance of 8 metres.***

*u = 0 (as it starts from rest)* $a = 0.5\ ms^{-2}$ $s = 8\ m$ $t = ?$

*Use* $s = ut + \frac{1}{2}at^2$

$8 = 0t + \frac{1}{2}(0.5)t^2$

$8 = 0.25t^2$

$32 = t^2$

$t = \sqrt{32} = 5.66$ s (3 SF)

**Vertical motion under gravity**

If we assume that air resistance can be ignored, then any body moving under gravity has acceleration *g* downwards, where $g = 9.81\ ms^{-2}$.

Since *g* is constant (for bodies moving close to the earth's surface, this is a good approximation), the equations of motion can be used. The same strategy as before should be used, but in addition, the following should be borne in mind:

- Direction is important. You should always decide which direction you are taking as positive at the start of the problem. You may find it helpful to **always** take **upwards** as **positive**
- The acceleration will always be *g* downwards – or –*g*, if you are taking upwards as positive
- The body will carry on going upwards until **v = 0**
- When the body returns to the ground, **s = 0**

> ***Exam Hint:-*** *Here are some common mistakes:*
> - *Thinking that v = 0 when the body returns to the ground – it isn't!*
> - *Thinking that a = 0 when the body is at its highest point – a doesn't change!*
> - *Ignoring directions – always ask yourself whether a displacement or velocity is up or down, and so whether it should be put as positive or negative.*
> - *Not using the value of g given in the question – you may be told to take it as 9.8, 9.81 or $10ms^{-2}$ – and you* ***must*** *use the value you are given.*

***Example 4. A ball is thrown vertically upwards with speed 20 $ms^{-1}$. Taking g = 10 $ms^{-2}$, calculate***
***a) Its velocity after 1.5 seconds.***
***b) The height to which it rises***
***c) The time taken for it to return to the ground.***

*We will take upwards as positive*

a) $u = 20\ ms^{-1}$ $a = -10ms^{-2}$ $t = 1.5s$ $v = ?$

*So we use* $v = u + at$

$v = 20 + (-10)(1.5)$

$v = 5ms^{-1}$

b) $u = 20\ ms^{-1}$ *v = 0 (as we want the highest point)* $a = -10ms^{-2}$

$s = ?$

*So we use* $v^2 = u^2 + 2as$

$0 = 20^2 + 2(-10)(s)$

$0 = 400 - 20s$

$20s = 400$

$s = 20$ m.

c) $u = 20\ ms^{-1}$ *s = 0 (since it has returned to ground)* $a = -10ms^{-2}$

$t = ?$

*So we use*

$s = ut + \frac{1}{2}at^2$

$0 = 20t + \frac{1}{2}(-10)t^2$

$0 = 20t - 5t^2$

$0 = 5t(4 - t)$

*So t = 0 or 4*

*We want t = 4, since t=0 is when particle was thrown upwards.*

NB: If you are unhappy about the factorising of $20t - 5t^2$ used above, then consult Factsheet 15 Maths for Physics: Algebraic Manipulation.

***Example 5. A child throws a stone vertically upwards from the top of a cliff with speed 15 $ms^{-1}$. Five seconds later, it hits the sea below the cliff. Taking g = 9.8 $ms^{-2}$, calculate***
***a) the velocity of the stone when it hits the sea***
***b) the height of the cliff***

*We will take upwards as positive*

a) $u = 15\ ms^{-1}$ $a = -9.8ms^{-2}$ $t = 5\ s$ $v = ?$

*So use* $v = u + at$

$v = 15 + (-9.8)(5)$

$v = 15 - 49 = -34ms^{-1}$

*So its velocity is 34 $ms^{-1}$ downwards (because of the minus sign)*

b) $u = 15ms^{-1}$ $a = -9.8ms^{-2}$ $t = 5\ s$ $s = ?$

*So use* $s = ut + \frac{1}{2}at^2$

$s = 15(5) + \frac{1}{2}(-9.8)(5)^2$

$s = 75 - 122.5 = -47.5$ m

*So the sea is 47.5 m below the cliff (minus sign – and we'd expect this!)*

*So the height of the cliff is 47.5 metres*

### Derivation of the Equations of Motion

Some exam boards require you to derive the equations of motion. If yours does, you must learn the following:

1. We know $a = \frac{\text{change in velocity}}{\text{time}} = \frac{v - u}{t}$

   So, multiplying up: $at = v - u$

   So, rearranging: $u + at = v$

2. We know average velocity = $\frac{\text{displacement}}{\text{time}}$

   But average velocity = $\frac{1}{2}(u + v)$

   So $\frac{1}{2}(u + v) = \frac{s}{t}$

   So $\frac{1}{2}(u + v)t = s$

3. From 1, we know $v = u + at$.

   Substituting this into $s = \frac{1}{2}(u + v)t$, we get:

   $s = \frac{1}{2}(u + u + at)t$

   $s = \frac{1}{2}(2u + at)t$

   $s = (u + \frac{1}{2}at)t$

   $s = ut + \frac{1}{2}at^2$

4. From 1, by rearranging we know $t = \frac{v - u}{a}$

   Substituting this into $s = \frac{1}{2}(u+v)t$, we get:

   $s = \frac{1}{2}(u + v)\frac{(v - u)}{a}$

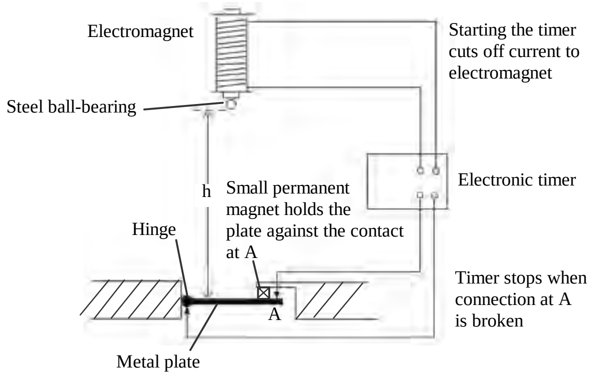
   $s = \frac{(u + v)}{2}\frac{(v - u)}{a} = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$

   $2as = v^2 - u^2$

**Typical Exam Question:**
A rocket accelerates from rest for 20s with a constant upward acceleration of $10ms^{-2}$. At the end of 20s the fuel is used up and it completes its flight under gravity alone. Assuming that air resistance can be neglected and taking $g = 9.8ms^{-2}$, calculate the:
(a) speed reached after 20s. [2]
(b) height after 20s. [2]
(c) maximum height reached. [3]
(d) speed just before the rocket hits the ground. [2]

*Taking upwards as positive:*
(a) $u = 0$, $a = 10ms^{-2}$, $t = 20$ s, $v = ?$
$v = u + at$
$v = 0 + 10(20)$ ✓ $= 200ms^{-1}$ ✓
(b) $u = 0$, $a = 10ms^{-2}$, $t = 20$ s, $s = ?$
$s = ut + ½at^2$
$s = 0(20) + ½ (10)(20^2)$ ✓
$s = 2000$ m ✓
(c) *Need to find height reached while moving under gravity. So start from point where fuel runs out.*
$u = 200ms^{-1}$ (from (a)) $a = -9.8 ms^{-2}$ $v = 0$ $s = ?$
$v^2 = u^2 + 2as$
$0 = 200^2 + 2(-9.8)(s)$ ✓
$s = \frac{200^2}{2 \times 9.8} = 2040m$ (3 SF) ✓
So total height = 2000 + 2040 = 4040m (3 SF) ✓
(d) *Starting from highest point:*
$s = -4040m$ $a = -9.8 ms^{-2}$ $u = 0$ $v = ?$
$v^2 = u^2 + 2as$
$v^2 = 2(-4040)(-9.8) = 79184$ ✓
$v = 281 ms^{-1}$ (3 SF) ✓

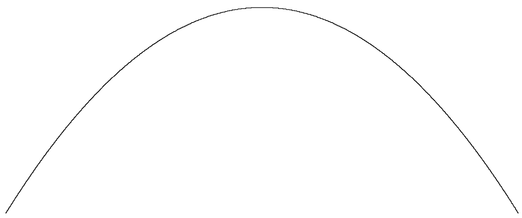
**Experimental Determination of *g***
This experiment determines *g* by finding the time a ball-bearing takes to fall from rest through a measured distance. The apparatus is shown below.




Since $u = 0$, we have $h = ½ gt^2$, so $g = \frac{2h}{t^2}$

To ensure reasonable accuracy,
- the timer must be accurate to 0.01 seconds.
- h is measured from the bottom of the ball-bearing, so the size of the ball-bearing does not introduce errors.
- the experiment should be repeated a number of times and the average found.
- the current in the electromagnet should be reduced to the minimum that will hold the ball-bearing. This reduces the chance of a delay in the ball-bearing being released.

**3. Projectiles – motion in two dimensions under gravity**
If you throw an object, it follows a **parabolic path** (shown below).



To deal with this situation, we consider the horizontal and vertical components of the motion **seperately**. Again, we assume that air resistance can be neglected
- **horizontally**, there is **no resultant force** on the object, so **its velocity is constant** (and there is **no acceleration**)
- **vertically,** gravity is the only resultant force. So its acceleration is ***g* downwards**

The general approach to problems is very similar to that used in the previous section, but the following should be borne in mind:
- In each part of the question, you must decide whether you need to use the horizontal or the vertical motion
- Again, take care with directions and signs
- At the highest point, v = 0 vertically
- If it returns to the same level at which it started, s = 0 vertically.
- To find its velocity, you need to find the **resultant** of its vertical and horizontal speeds (see Factsheet 02 Vectors & Forces )
- If it is thrown at a speed U and angle α to the horizontal, then
  - the horizontal component of velocity is $U\cos\alpha$
  - the vertical component of velocity is $U\sin\alpha$

  *(NB: some exam boards only consider bodies projected horizontally or vertically, rather than at an angle).*

***Example 1. A vase is thrown out of a first floor window, which is 5 m above the ground, with a horizontal velocity of $4ms^{-1}$.***
***Taking $g = 10ms^{-2}$, find***
***a) The time taken for the vase to hit the ground***
***b) The horizontal distance it travels.***
***c) Its speed as it hits the ground.***

*We will take upwards as positive*
a) *Since this involves the **vertical** position of the vase, we must consider the vertical motion*

*Vertically:* $u = 0$ (since thrown horizontally) $a = -10ms^{-2}$
$s = -5m$ (since it is going downwards) $t = ?$
Using $s = ut + ½ at^2$:
$-5 = 0(t) + ½ (-10)t^2$
$-5 = -5t^2$
$t = 1$ second.

b) *We must consider horizontal motion*
$u = 4 ms^{-1}$ $a = 0$ $t = 1$ s (from a)) $s = ?$

Using $s = ut + ½ at^2$:
$s = 4(1) = 4m$.

c) *This is the magnitude of its velocity.*
*We need both horizontal and vertical components.*
*Vertically:* $u = 0$ $a = -10ms^{-2}$ $t = 1s$ (from a))
$v = u + at$
$v = 0 + (-10)(1) = -10ms^{-1}$

*Horizontally, velocity is constant, so* $v = 4ms^{-1}$

*Resultant speed* $= \sqrt{(-10)^2 + 4^2} = 10.8 ms^{-1}$

***Example 2.A ball is thrown from ground level with a speed of 20 ms$^{-1}$ at an angle of 30$^o$ to the horizontal. Taking g = 9.8ms$^{-2}$, find:***
***a) The greatest height it reaches***
***b) The time taken for the ball to return to ground level.***
***c) The horizontal distance the ball travels in this time.***

*Take upwards as positive:*

a) *This is vertical motion.*
$u = 20\sin30^o\ ms^{-1}$ $\quad a = -9.8\ ms^{-2}$ $\quad v = 0$ $\quad s = ?$

$2as = v^2 – u^2$
$2(-9.8)s = 0 – (20\sin30^o)^2$
$-19.6s = -100$
$s = 100 \div 19.6 = 5.10\ m$

b) *Again, vertical motion as its level is referred to.*
$u = 20\sin30^o\ ms^{-1}$ $\quad a = -9.8ms^{-1}$ $\quad s = 0$ $\quad t = ?$

$s = ut + ½ at^2$
$0 = 20\sin30^o t + ½ (-9.8)t^2$
$0 = 10t – 4.9t^2$
$0 = t(10 – 4.9t)$
$t = 0$ *(not applicable) or* $10\div4.9$
*So* $t = 2.04$ *s (3 SF)*

c) *Horizontal motion:*
$u = 20\cos30^o ms^{-1}$ $\quad a = 0$ $\quad t = 2.04$ *s (from b))* $s = ?$

$s = ut + ½ at^2$
$s = 20\cos30^o\ (2.04) = 35$ *m (2 SF)*

**Questions**

1. Explain the difference between velocity and speed

2. A particle is moving in a straight line with constant acceleration. It passes through point A with speed 4ms$^{-1}$, and 2 seconds later, through point B with speed 3ms$^{-1}$. Find
 a) Its acceleration
 b) The distance between A and B

3. A ball is thrown vertically upwards from ground level with speed 25ms$^{-1}$. Taking $g$ = 10ms$^{-2}$, find
 a) Its speed when it is 2m above ground level
 b) Its greatest height

4. A ball is thrown horizontally from the top of a cliff with speed 10ms$^{-1}$, and later falls into the sea. The cliff is 50m high. Taking $g$ = 10ms$^{-2}$, find
 a) The time taken for the ball to reach the sea
 b) The distance from the bottom of the cliff that it lands
 c) The magnitude and direction of its velocity as it reaches the sea

5. A ball is thrown from ground level with a speed of 28.2 ms$^{-1}$ at an angle of 45$^o$ to the horizontal. Taking $g$ = 9.81ms$^{-2}$, find
 a) The time it takes to reach its greatest height
 b) The time taken to travel a horizontal distance of 30m

There is a tree 30 m from the point from which the ball was thrown. The ball just passes over the top of it.
 c) Find the height of the tree

**Answers**
1. See page 1
2. a) a= -0.5 ms$^{-2}$ b) 7 m
3. a) 24 ms$^{-1}$ (2 SF) b) 31 m (2 SF)
4. a) 3.2 s (2 SF) b) 32 m (2 SF)
 c) 34ms$^{-1}$ 73$^o$ below horizontal
5. a) 2.03 s (3 SF) b) 1.50 s (3 SF) c) 18.9 m

## Exam Workshop

**This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.**

**A dart player throws a dart horizontally. By the time it reaches the dartboard, 3.00m away, it has fallen a height of 0.200m. Taking g as *9.81ms*** $^{-2}$ **, find:**
**(a) The time of flight [2]**
$s = ½ gt^2$
$0.2 = 5t^2$ ✓
$t = 0.2\ s$ ✗

Examiner's comment: The candidate has used a correct method, but has used $g$ = 10ms$^{-2}$. Read the question!

**(b) The initial velocity [2]**
$3 = u0.2$ ✓
$u= 15ms^{-1}$

Examiner's comment: Full marks would have been awarded here as a "follow through" from the error in part a), but for the fact the candidate has not given the direction of the velocity, just its magnitude.

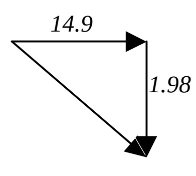
**(c) The magnitude and direction of the velocity as it is just about to hit the dartboard. [6]**
$v = gt = 2\ ms^{-1}$ ✓✓
$15 + 2 = 17ms^{-1}$ ✗

Examiner's comment: The candidate started correctly, by considering the horizontal and vertical components of the velocity. However, the candidate does not seem to appreciate that the two components must be combined as vectors. No attempt has been made to find the direction – the fact that this was asked for should have alerted the candidate to the inadequacy of his/her method.

***Examiner's Answers***
a) *Vertical:* $s = -0.2\ m$ $\quad a = - 9.81ms^{-2}$ $\quad u = 0$ $\quad t = ?$
$s = ut + ½ at^2$
$-0.2 = -4.905t^2$ ✓
$t = 0.202\ s$ ✓*(3 SF)*

b) *Horizontally:* $s = 3m$ $\quad a = 0$ $\quad t = 0.202s$ $\quad u = ?$
$s = ut + ½ at^2$
$3 = u(0.202)$ ✓
$u = 14.9\ ms^{-1}$ *(3 SF)* ✓

c) *Vertically:* $u = 0$ $\quad t =0.202\ s$ $\quad a = -9.81\ ms^{-2}$
$v = u + at$
$v = (-9.81)(0.202)$ ✓$= -1.98\ ms^{-1}$ ✓




*magnitude =* $\sqrt{(14.9^2 + 1.98^2)}$ ✓$= 15\ ms^{-1}$ ✓*(2 SF)*
*at angle* $\tan^{-1}(1.98/14.9)$ ✓$= 7.6^o$ *(2SF) below horizontal* ✓

***Acknowledgements:*** *This Factsheet was researched and written by Cath Brown*
***Curriculum Press, Unit 305B The Big Peg, 120 Vyse Street, Birmingham B18 6NF.***
***Physics Factsheets*** *may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber. They may be networked for use within the school. No part of these Factsheets may be reproduced, stored in a retrieval system or transmitted in any other form or by any other means without the prior permission of the publisher. ISSN 1351-5136*