FUNCTIONS

2.1.

A function f is defined by $f: x \to \dfrac{e^x + 1}{4}$ for the domain $x \geqslant 0$.

(i) Evaluate $f^2(0)$. [3]

(ii) Obtain an expression for f^{-1}. function [2]

(iii) State the domain and the range of f^{-1}. [2]

(i)	f(0) = ½ $f^2(0) = f(½) = (\sqrt{e} + 1)/4 \approx 0.662$ (accept 0.66 or better)	B1 M1 A1
(ii)	$x = (e^y + 1)/4$ $\Rightarrow e^y = 4x - 1$ $\Rightarrow f^{-1} : x \mapsto \ln(4x - 1)$	M1 A1
(iii)	Domain of f^{-1} is $x \geq ½$ Range of f^{-1} is $f^{-1} \geq 0$	B1 B1
[7]		

2.2

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto e^x,$$

$$g : x \mapsto 2x - 3.$$

(i) Solve the equation $fg(x) = 7$. [2]

Function h is defined as gf.

(ii) Express h in terms of x and state its range. [2]

(iii) Express h^{-1} in terms of x. [2]

(i) e^{2x-3} (= 7) $\Rightarrow$ x = ½ (3 + ln 7) ≈ 2.47 ~ 2.48 (not 2.5)	M1 A1
(ii) h = $2e^x - 3$ (x, y or) h > – 3 accept ≥	B1 B1
(iii) h^{-1} (or y) = ln {½ (x + 3)} or ln(x + 3) – ln2 or lg{½(x + 3)}/lge but ln{½(y + 3)} M1 A0 lg (or log) {½(x + 3)} M1 A0	M1 A1 (M1 for logs taken in valid way

2.3

Express $6 + 4x - x^2$ in the form $a - (x + b)^2$, where a and b are integers. [2]

(i) Find the coordinates of the turning point of the curve $y = 6 + 4x - x^2$ and determine the nature of this turning point. [3]

The function f is defined by $\mathrm{f} : x \mapsto 6 + 4x - x^2$ for the domain $0 \leqslant x \leqslant 5$.

(ii) Find the range of f. [2]

functions

(iii) State, giving a reason, whether or not f has an inverse. [1]

[8]	$6 + 4x - x^2 \equiv 10 - (x - 2)^2$	M1 A1
	(i) $x = 2$ $y = 10$ Maximum	B1√B1√B1
	(ii) f(0) = 6, f(2) = 10, f(5) = 1 $\Rightarrow$ $1 \le f \le 10$ [alternatively 1 ≤ B1, ≤ 10 B1]	M1 A1
	(iii) f has no inverse; it is not 1:1	B1

2.4

(i) Sketch the graph of $y = |3x + 9|$ for $-5 < x < 2$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = x + 6$. [1]

(iii) Solve the equation $|3x + 9| = x + 6$. [3]

(i)	Idea of modulus correct	M1	
	Shape and position completely correct	A1	
	(0, 9) (–3, 0) indicated on graph	A1	
(ii)	Straight line with +ve gradient and +ve y intercept, correct position	B1	
(iii)	$3x + 9 = x + 6 \Rightarrow x = -1.5$	B1	
	Solve $-(3x + 9) = (x + 6)$ or $(3x + 9)^2 = (x + 6)^2$	M1	
	$x = -3.75$	A1	[7]