

# FUNCTIONS

**1**

(a) It is given that  $f(x) = \frac{1}{2+x}$  for  $x \neq -2, x \in \mathbb{R}$ . [2]

(i) Find  $f''(x)$ .

(ii) Find  $f^{-1}(x)$ . [2]

(iii) Solve  $f^2(x) = -1$ . [3]

(b) The functions  $g$ ,  $h$  and  $k$  are defined, for  $x \in \mathbb{R}$ , by

$$g(x) = \frac{1}{x+5}, \quad x \neq -5,$$

$$h(x) = x^2 - 1,$$

$$k(x) = 2x + 1.$$

Express the following in terms of  $g$ ,  $h$  and/or  $k$ .

(i)  $\frac{1}{(x^2-1)+5}$

[1]

(ii)  $\frac{2}{x+5} + 1$

[1]

-----Marking Scheme-----

<p><b>(a) (i)</b> <math>f'(x) = -(2+x)^{-2}</math>  <math>f''(x) = 2(2+x)^{-3}</math></p>	<p><b>B1</b> First <b>B1</b> may be implied by a correct answer for <math>f''(x)</math>  <b>B1</b> If done by quotient rule, allow unsimplified</p>
<p><b>(ii)</b> <math>y = \frac{1}{2+x}, x = \frac{1}{y} - 2</math>  <math>f^{-1}(x) = \frac{1}{x} - 2</math> or <math>\frac{1-2x}{x}</math></p>	<p><b>M1</b> <b>M1</b> for a valid attempt at the inverse  <b>A1</b> <b>A1</b> must be in correct form, allow <math>y = \dots</math></p>
<p><b>(iii)</b> <math>f^2(x) = \left( \frac{1}{2 + \frac{1}{2+x}} \right) = \frac{2+x}{5+2x}</math>            Equating to <math>-1</math> leads to <math>x = -\frac{7}{3}</math> or <math>-2.33</math></p>	<p><b>M1</b> <b>M1</b> for correct attempt at <math>f^2(x)</math>  <b>DM1</b> <b>DM1</b> for attempt at solution of <math>f^2(x) = -1</math>  <b>A1</b> <b>A1</b> for <math>x = -\frac{7}{3}</math> or equivalent</p>
<p><b>(b) (i)</b> <math>gh(x)</math> or <math>gh</math></p>	<p><b>B1</b> <b>B1</b> for either form</p>
<p><b>(ii)</b> <math>kg(x)</math> or <math>kg</math></p>	<p><b>B1</b> <b>B1</b> for either form</p>
	<p><b>[9]</b></p>

## 2

(a) Functions  $f$  and  $g$  are defined, for  $x \in \mathbb{R}$ , by

$$f(x) = 3 - x,$$
$$g(x) = \frac{x}{x+2}, \text{ where } x \neq -2.$$

(i) Find  $fg(x)$ . [2]

(ii) Hence find the value of  $x$  for which  $fg(x) = 10$ . [2]

(b) A function  $h$  is defined, for  $x \in \mathbb{R}$ , by  $h(x) = 4 + \ln x$ , where  $x > 1$ .

(i) Find the range of  $h$ . [1]

(ii) Find the value of  $h^{-1}(9)$ . [2]

(iii) On the same axes, sketch the graphs of  $y = h(x)$  and  $y = h^{-1}(x)$ . [3]

<p><b>(a) (i)</b> <math>fg(x) = f\left(\frac{x}{x+2}\right)</math>  <math>= 3 - \frac{x}{x+2}</math></p>	<p>M1 A1 [2]</p>	<p>M1 for order</p>
<p><b>(ii)</b> <math>3 - \frac{x}{x+2} = 10</math>                      leading to <math>x = -1.75</math></p>	<p>DM1 A1 [2]</p>	<p>DM1 for dealing with fractions sensibly</p>
<p><b>(b) (i)</b> <math>h(x) &gt; 4</math></p>	<p>B1 [1]</p>	
<p><b>(ii)</b> <math>h^{-1}(x) = e^{x-4}</math>  <math>h^{-1}(9) = e^5 \quad (\approx 148)</math>                      or <math>4 + \ln x = 9</math>,                      leading to <math>x = e^5</math></p>	<p>M1 A1 [2]</p>	<p>M1 for attempting to obtain inverse function</p>
<p><b>(iii)</b> correct graphs</p>	<p>B1 B1 B1 [3]</p>	<p>B1 for each curve B1 for idea of symmetry</p>